Lec 5 Digital Control

1. using Jury test, check the system stability for:

$$Z^3 - Z^2 - 0.25Z + 0.25 = 0$$

2. For the following system; find range of K for stability.

$$\overline{GH}(z) = \frac{KZ(2Z-1)}{(Z-1)(Z-0.8)}$$

← رقم (1) حلها عادي.

if system order (2) → use Jury or Routh.

if system order > 2 → you better use Routh.

2) ch. equation $1 + \overline{GH}(z) = 0$

$$1 + \frac{KZ(2Z-1)}{(Z-1)(Z-0.8)} = 0$$

$$(Z-1)(Z-0.8) - KZ(2Z-1) = 0$$

$$\boxed{\underbrace{(1+2K)}_{a_2}{}^2 + (-1.8-K)Z + \underbrace{0.8}_{a_0} = 0}$$

using Jury test

1) $F(1) > 0$

$1 + 2K - 1.8 - K + 0.8 > 0$

$\boxed{K > 0} \longrightarrow (1)$

2) $(-1)^2 F(-1) > 0$

$1 + 2K + 1.8 + K + 0.8 > 0$

$3K > -3.6 \Rightarrow \boxed{K > -1.2} \longrightarrow (2)$

3) $|a_0| = 0.8 < |a_2| = 1 + 2K$

$\boxed{K > -0.1} \longrightarrow (3)$

$\Rightarrow$ Range of stability $\boxed{K > 0}$

← في $(n = 2)$ عندما تستخدم (Jury) لإيجاد

(range of stability) هتستخدم أول ثلاثة شروط فقط

و التقاطع ما بينهم هو الحل . لكن $(n > 2)$ استخدم

(Routh) لإيجاد Range of K.

Report

$$\overline{GH(z)} = \frac{K(Z-0.2)}{(z-1)(z+0.6)^2}$$

Find the range of K for stability using:

1) bilinear transformation

2) Jury test

Root locus

stability

1) Relative stability

"to what range sys. is stable"

~~[illegible]~~

- GM & PM "stability indicators"
  - Bode diagram.
  - Polar Plot
  - Nyquist

2) absolute stability

→ Tell us the stability of sys. as (stabe, unstable, critically)

- Jury
- bilinear transformation (Routh)

stability

1) Graphical methods

- Root locus.
- Bode diagram
- Polar Plot
- Nyquist

2) Algebric Methods

→ Jury test

→ Routh array "using bilinear transformation"

$\boxed{EX1}$ $\overline{GH(z)} = \frac{K(z+1)}{(1-z)^2} = \frac{K(z+1)}{(z-1)^2}$

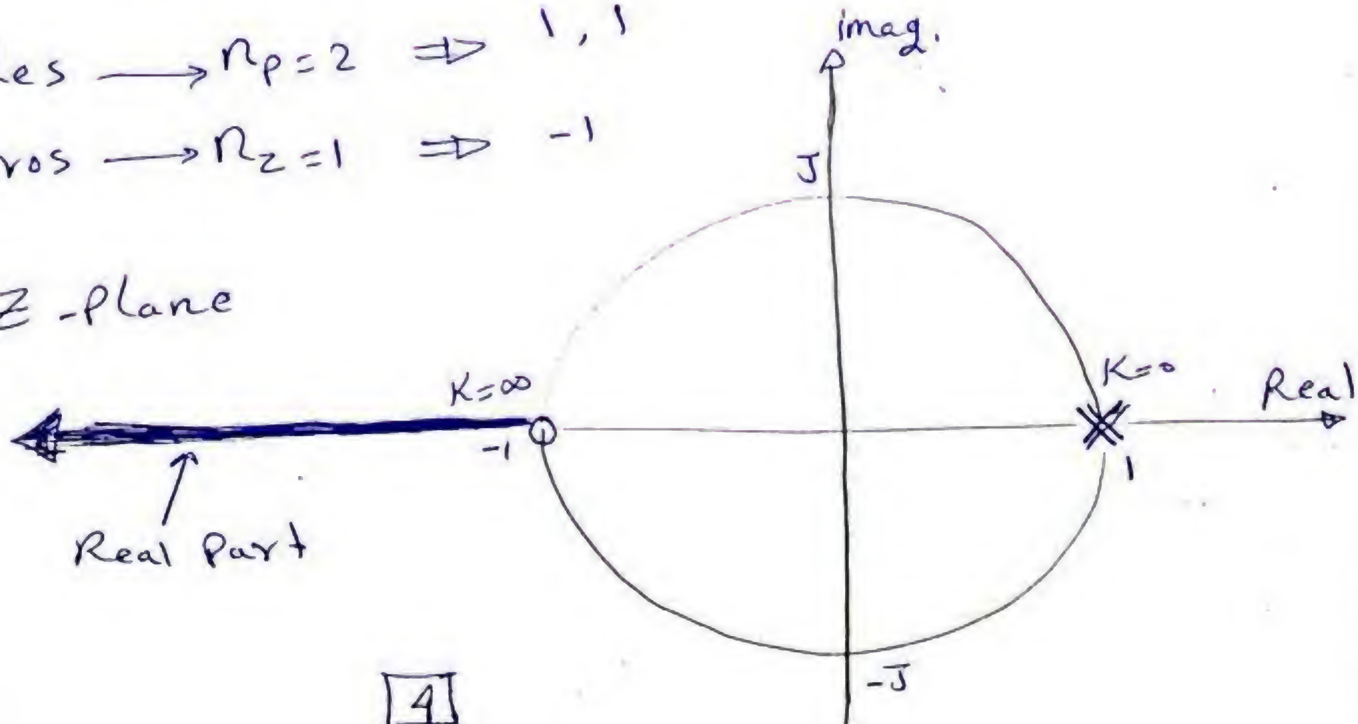
→ Draw the root locus & find the critical value for K (K_{cr})

← هنا بندرس حركة ال (Poles) بالنسبة لدائرة الوحدة.

1) Poles → $n_p = 2$ ⇒ 1, 1

Zeros → $n_z = 1$ ⇒ −1

2) Z-plane



3) Real Part

from -1 to $-\infty$

4) Asymptotes (مش مفيدة هنا)

a) no. of Asym. $= n_p - n_z = 2 - 1 = 1$

b) C_A or $\beta_A = \dfrac{\sum \text{Poles} - \sum \text{zeros}}{n_p - n_z} = 3$

c) $\theta = \dfrac{(2L+1)180}{n_p - n_z}$ $\quad \theta_1 = 180$

← ينتج خط زاويته (180) مش مفيد هنا (كان للتوضيح فقط)

5) Breaking Point

ch. equation $\quad 1 + GH(z) = 0$

$$1 + \frac{K(Z+1)}{(Z-1)^2} = 0$$

$$(Z-1)^2 + K(Z+1)$$

$$K = -\frac{(Z-1)^2}{(Z+1)} = -\frac{Z^2 - 2Z + 1}{Z+1}$$

$$\frac{dK}{dz} = 0$$

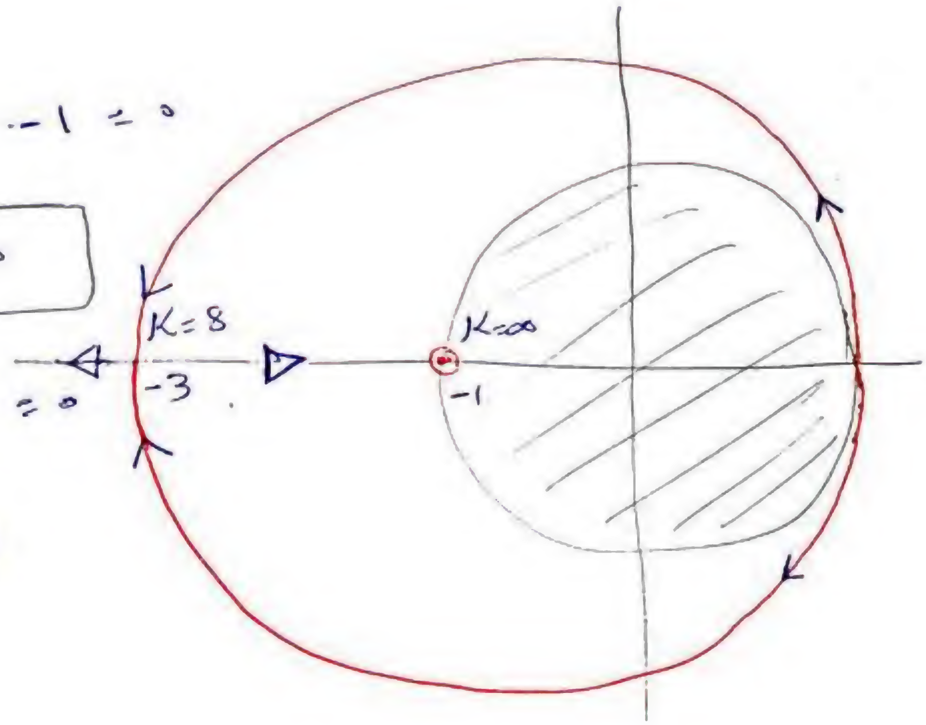
$$\frac{dK}{dz} = -\left[\frac{(z+1)(2z-2) - (z^2-2z+1)(1)}{(z+1)^2}\right] = 0$$

$$2z^2 - 2 - z^2 + 2z - 1 = 0$$

$$\boxed{z^2 + 2z - 3 = 0}$$

$$(z-1)(z+3) = 0$$

$$z = 1 \quad , \quad z = -3$$



Breaking Points

1) at $z=1$ (Breakaway Point)

$$K\Big|_{z=1} = 0$$

2) at $z=-3$ (Breakin Point

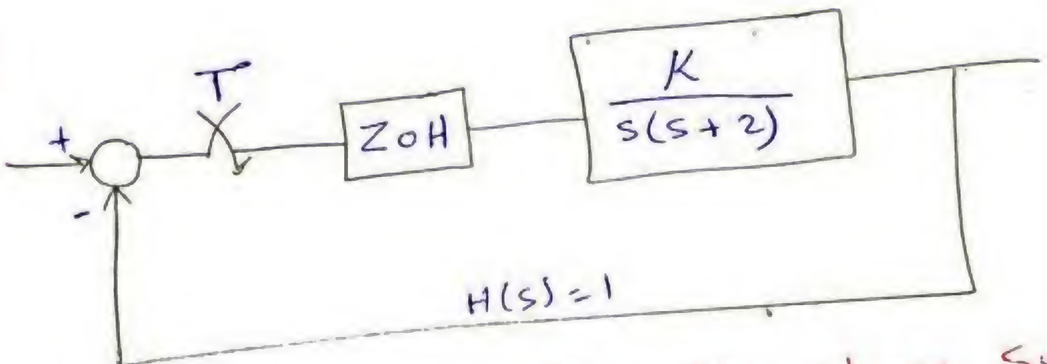
$$K\Big|_{z=-3} = 8$$

← نصف قطر الدائرة ← 2
← مركز الدائرة عند ← -1

→ Range of critically stable

↳ system is unstable for all $K > 0$

EX



- Draw the root locus for the above system for $T = 0.4$ & $T = 3$ sec and find K_{cr}

→ open loop T.F

$$\overline{GH}(z) = G(z) = Z\left[\frac{(1-e^{-Ts})K}{s^2(s+2)}\right]$$

$$= K(1-z^{-1})\, Z\left[\frac{1}{s^2(s+2)}\right]$$

$$= K(1-z^{-1})\, Z\left[\frac{A_1}{s^2} + \frac{A_2}{s} + \frac{A_3}{s+2}\right]$$

$$= K(1-z^{-1})\, Z\left[\underbrace{\frac{0.5}{s^2} - \frac{0.25}{s} + \frac{0.25}{s+2}}_{\mathcal{L}^{-1}\cdot T}\right]$$

$$\overline{GH}(z) = K\left(\frac{z-1}{z}\right) Z\left(0.5t\Big|_{t=KT} - 0.25 + 0.25e^{-2t}\Big|_{t=KT}\right)$$

$$= \frac{K}{4}\left(\frac{z-1}{z}\right) Z\left[(2t-1+e^{-2t})\Big|_{t=KT}\right]$$

$$= \frac{K}{4}\left(\frac{z-1}{z}\right)\left[\frac{2T\cancel{z}}{(z-1)^2} - \frac{\cancel{z}}{z-1} + \frac{\cancel{z}}{z-e^{-2T}}\right]$$

$$= \frac{K}{4}\left[\frac{2T}{z-1} - 1 + \frac{z-1}{z-e^{-2T}}\right]$$

$$= \frac{K}{4}\left[\frac{2T(z-e^{-2T}) - (z-1)(z-e^{-2T}) + (z-1)^2}{(z-1)(z-e^{-2T})}\right]$$

$$\overline{GH}(z) = \frac{K}{4}\left[\frac{(2T-1+e^{-2T})z + (1-2Te^{-2T}-e^{-2T})}{(z-1)(z-e^{-2T})}\right]$$

$$\boxed{T = 3 \text{ sec}} \qquad e^{-2T} = e^{-6} \simeq 0.0025 \simeq 0$$

$$\overline{GH(z)} = \frac{K}{4} * \frac{5Z+1}{Z(Z-1)}$$

$$= \underbrace{\frac{5}{4}K}_{\acute{K}} \frac{(Z+0.2)}{Z(Z-1)}$$

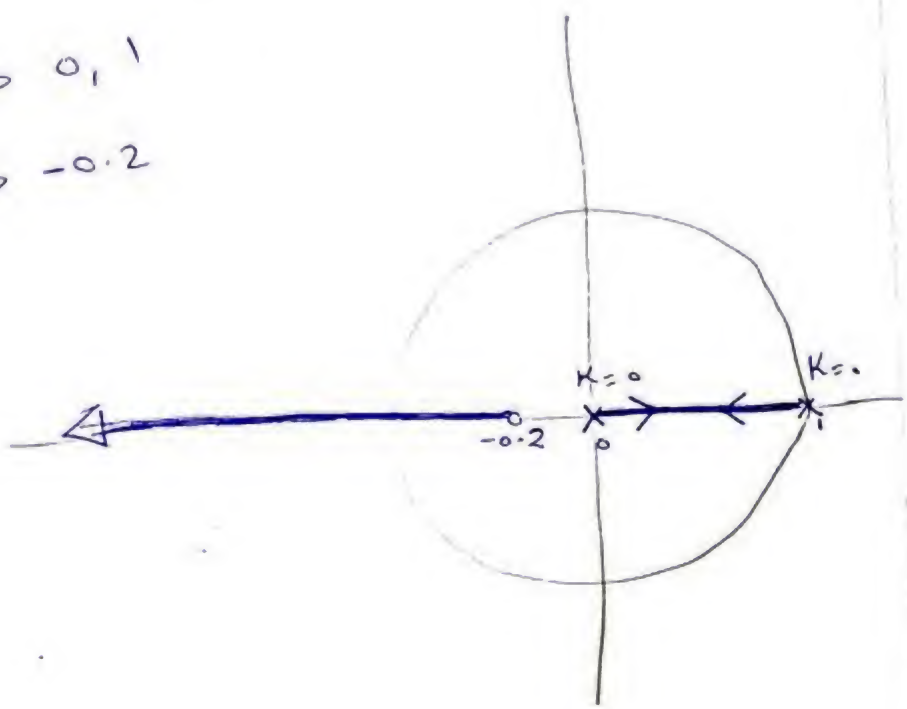
$; \acute{K} = 1.25\,K = \frac{5}{4}K$

$$\overline{GH(z)} = \frac{\acute{K}(Z+0.2)}{Z(Z-1)}$$

1) Poles : $n_p = 2 \Rightarrow 0, 1$

zeros : $n_z = 1 \Rightarrow -0.2$

2) Z - Plane



3) Real Part

from

$0 \longrightarrow 1$

$-0.2 \longrightarrow -\infty$

$\rightsquigarrow$ the most Place of stability is at the origin.

لـ كل أما أقرب للصفر بوصل لقمة ال (stability).

4) Asymptotes $\rightsquigarrow$ غير مفيدة.

5) Breaking Points

ch. equation $\Rightarrow 1+\overline{GH}(z)=0$

$$1+\frac{K'(z+0.2)}{z(z-1)}=0$$

$$z(z-1)+K'(z+0.2)$$

$$\boxed{K'=-\frac{z(z-1)}{z+0.2}}$$

$$\frac{dK'}{dz}=0$$

$$=-\left[\frac{(z+0.2)(2z-1)-z(z-1)(1)}{(z+0.2)^2}\right]$$

$$= -\left[\frac{2z^2 + 0.4z - z - 0.2 - z^2 + z}{(z+0.2)^2}\right] = 0$$

$$-\left(z^2 + 0.4z - 0.2\right) = 0$$

$$z_{1,2} = 0.29 \;,\; -0.69$$

Breaking Points

a) Breakaway Point at $z_1 = 0.29$

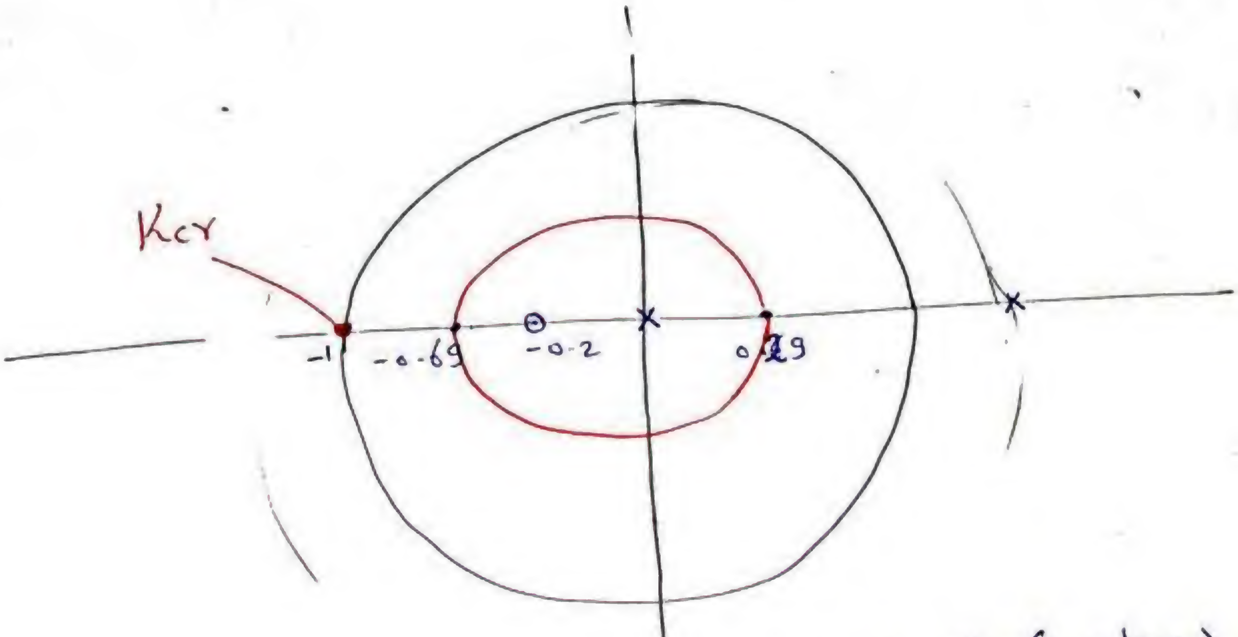
$$\acute{K}\Big|_{z_1 = 0.29} = 0.42 \Rightarrow \acute{K} = 1.25\,K$$

$$K = 0.336$$

b) Breakin Point at $z_2 = -0.69$

$$\acute{K}\Big|_{z_2 = 0.69} = 2.38 = 1.25\,K$$

$$K = 1.9$$

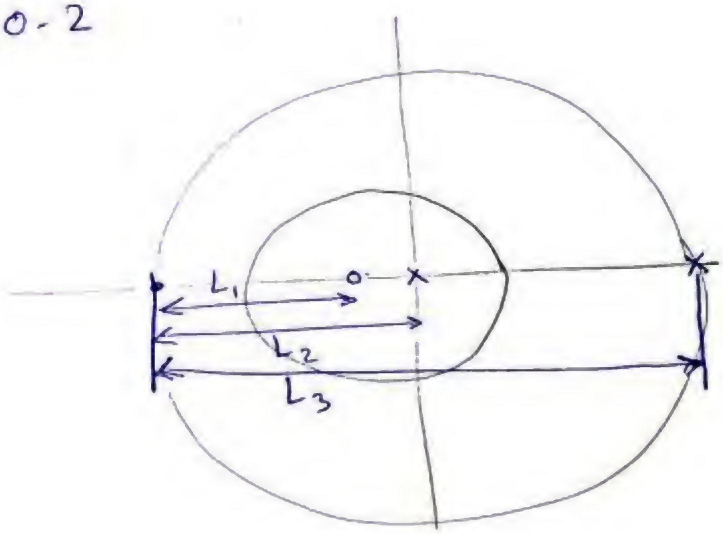


ال (system) يكون (stable)
حتى النقطة K_{cr}.
L at $Z = -1$

$$r = \frac{0.29 + 0.69}{2} = 0.49$$

$$C = 0.29 - 0.49 = -0.2$$

$\boxed{6}$ Determine K_{cr}



① $$K' = \frac{\Pi \text{ Poles}}{\Pi \text{ Zeros}}$$

$$= \frac{L_2 L_3}{L_1}$$

$$K' = \frac{1 * 2}{0.8} = 2.5 = 1.25K$$

$$\boxed{K = 2} \longrightarrow K_{cr}$$

range of stability
$0 < K < 2$

② another solution

$$\grave{K} = -\left[\frac{Z(Z-1)}{Z+0.2}\right]$$

$\rightarrow$ critical gain K_{cr} at $Z=-1$

$$\grave{K}_{cr} = -1\left[\frac{-1(-1-1)}{-1+0.2}\right] = 2.5 = 1.25\, K_{cr}$$

$$\boxed{K_{cr} = 2}$$

2] $T = 0.4$ sec $\rightarrow e^{-0.8} \simeq 0.45$

$$\overline{GH}(z) = \frac{K}{4}\left[\frac{0.25Z+0.19}{(Z-1)(Z-0.45)}\right]$$

$$= \frac{K}{16}\left[\frac{Z+\frac{0.19}{0.25}}{(Z-1)(Z-0.45)}\right] = \frac{K}{16}\left[\frac{Z+0.76}{(Z-1)(Z-0.45)}\right]$$

$$\grave{K} = \frac{K}{16}$$

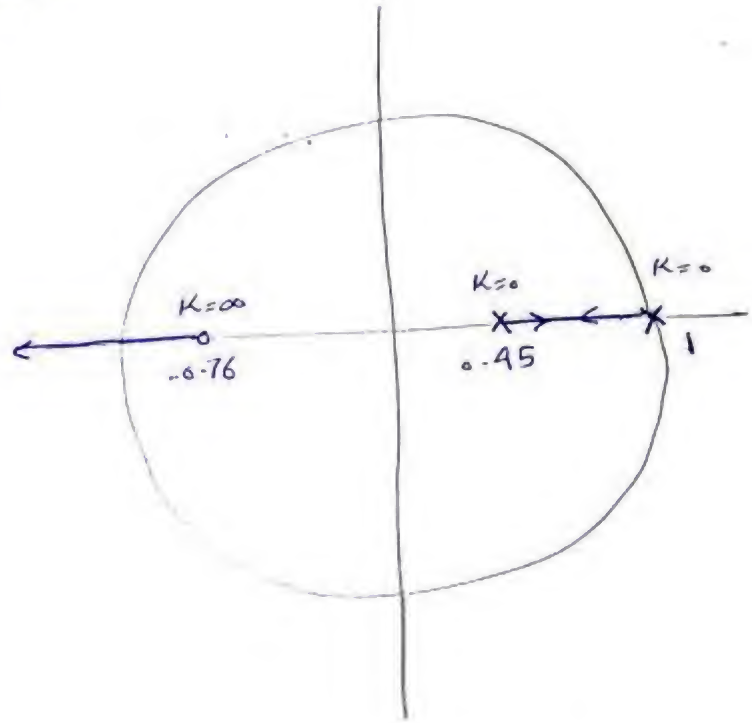
$$\overline{GH(z)} = K' \left(\frac{Z + 0.76}{(Z-1)(Z-0.45)} \right)$$

1)

Poles (2) $\Rightarrow$ 1, 0.45

zeros (1) $\Rightarrow$ −0.76

2) Z-Plane



3) Real Part

$0.45 \longrightarrow 1$

$-\infty \longrightarrow -0.76$

4) Breaking Point

ch. equation $\quad 1 + \overline{GH(z)} = 0$

$$1 + K' \frac{Z + 0.76}{(Z-1)(Z-0.45)} = 0$$

$$(Z-1)(Z-0.45) + K'(Z + 0.76) = 0$$

$$K' = -\left[\frac{(Z-1)(Z-0.45)}{Z + 0.76} \right]$$

$$\frac{d\acute{K}}{dz} = 0$$

$$\boxed{z^2 + 1.52\, z - 1.552 = 0}$$

$$z_{1,2} = 0.7 \;\&\; -2.22$$

Breaking Points

a) Break away Point $z_1 = 0.7$

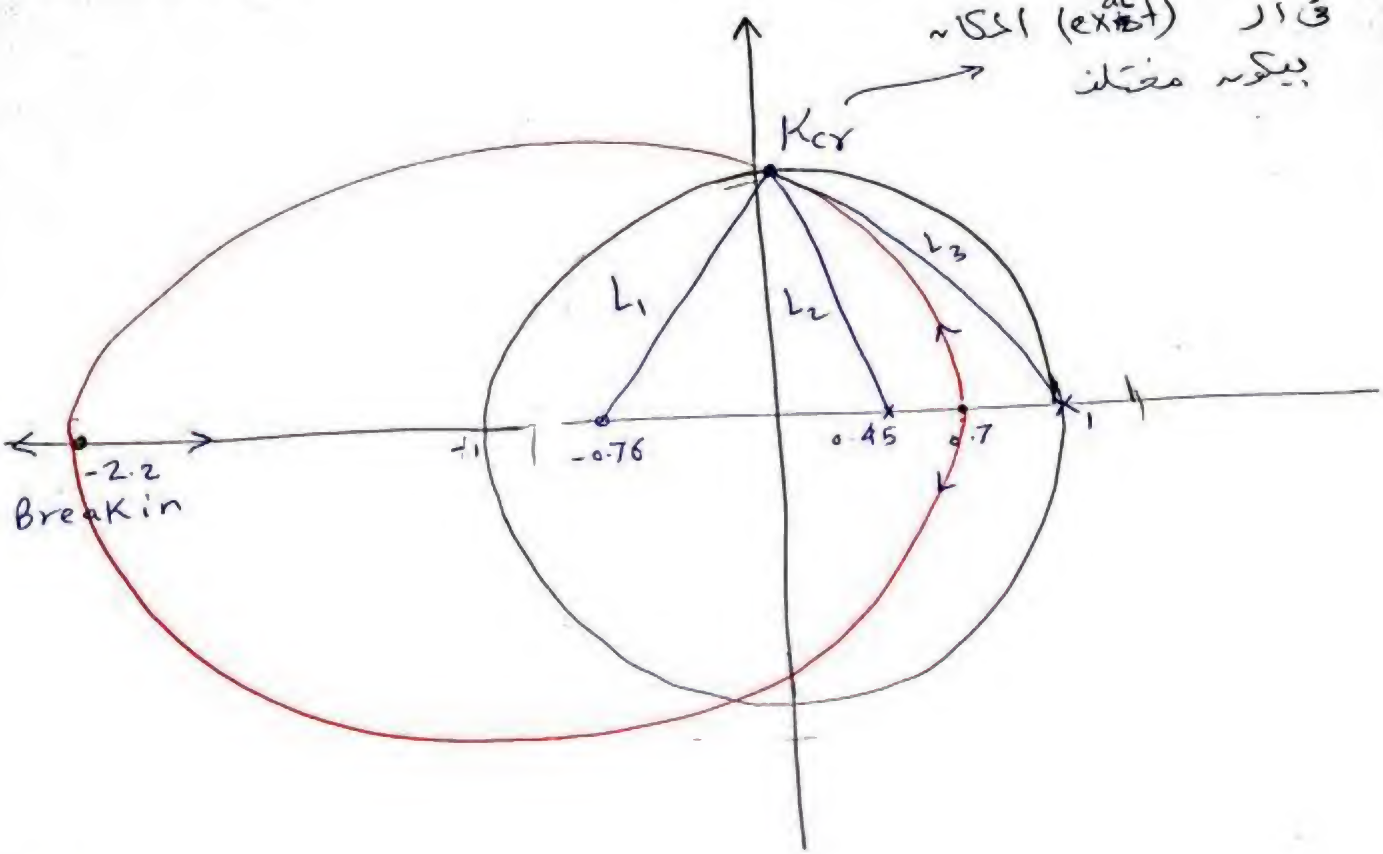
$$\acute{K}\Big|_{z_1=0.7} = 0.05137 = \frac{K}{16} \Rightarrow K = 0.822$$

b) Breakin Point $z_2 = -2.22$

$$\acute{K}\Big|_{z_2=-2.22} = 5.888 = \frac{K}{16} \Rightarrow K = 94.2$$

$$r = \frac{0.7 + 2.22}{2} = 1.46$$

$$c = 0.7 - 1.46 = -0.76$$



a)

$$\acute{K}_{cr} = \frac{L_2 \cdot L_3}{L_1} = \frac{K}{16} \Rightarrow K = \text{--}$$

دي أفضل في ال (exact).

b) using Jury test $\Rightarrow 1 + \overline{GH}(z) = 0$

$$1 + \acute{K}\,\frac{Z + 0.76}{(Z-1)(Z-0.45)} = 0$$

$$F(z) = Z^2 + (-1.45 + \acute{K})Z + (0.45 + 0.76\acute{K}) = 0$$

using Jury 1st 3 conditions $\Rightarrow 0 < \acute{K} < 0.7237$

$\rightarrow$ using

$$\boxed{0 < K_{cr} < 11.578}$$

c) ch.equation

$$\acute{K} = -\left[\frac{(Z-1)(Z-0.45)}{Z+0.76}\right]$$ المعامل

(معادلة أي دائرة)

$$(x-x_0)^2 + (y-y_0)^2 = r^2$$

center (x_0, y_0)

radius $\longrightarrow r$

① unit circle $x^2 + y^2 = 1 \longrightarrow (1)$

② circle of root locus

$$(x+0.76)^2 + y^2 = (1.46)^2 \longrightarrow (2)$$

← حلهم معاً والتقاطع ما بينهم ينتج $\acute{K}_{cr}$

from 1 in 2

$$x^2 + 1.52x + 0.5776 + 1 - x^2 = 2.1316$$

$$\boxed{x \simeq 0.364} \Rightarrow (0.364)^2 + y^2 = 1 \Rightarrow \boxed{y = \pm j0.93}$$

The critical gain $\grave{K}_{cr}$

at $\boxed{Z = 0.364 \pm J\,0.931}$

$$\grave{K} = \frac{(Z-1)(Z-0.45)}{(Z+0.76)}$$

$$|\grave{K}| = \left.\frac{|Z-1|\,|Z-0.45|}{|Z+0.76|}\right|_{Z=0.364 \pm J\,0.931}$$

$$\simeq 0.7223 = \frac{K}{16}$$

$$\boxed{K_{cr} = 11.56}$$